

Stability analysis and optimization of gravity retaining wall using Matlab

Gayathri N P, Prof.Mariamamma Joseph

Abstract—Retaining walls are generally used to maintain the ground surfaces at different elevations. Among various types of retaining walls, gravity retaining walls are commonly used. It depends on self weight for its stability. All structures should be checked for safety conditions. Also optimization of retaining wall is another important task in its design. It is very difficult to design a structure considering safety conditions and optimum cost and weight. This can be easily done by adopting a non linear programming technique. MATLAB can be used for this purpose which is a high performance language for technical computing. Here a program is developed in MATLAB to analyse the stability of gravity retaining wall and optimization is done using OPTIMTOOL in MATLAB. The program considers proportioning of retaining wall, its self-weight, soil type and its conditions etc. Validation by considering an existing retaining wall is also done.

Index Terms—MATLAB, Optimization, Optimtool, Proportioning, Retaining wall, Stability, Validation

1 INTRODUCTION

1.1 General

Rigid walls which are used to support the soil mass laterally are generally called as retaining walls. It helps to retain the soil at different levels on the two sides. They are also used in areas where there is a need of shaping the landscape severely and to be engineered for more purposes like hillside farming, roadway overpasses etc. Different types of retaining walls existing are gravity retaining walls, semi gravity retaining walls, cantilever retaining walls, counterfort retaining walls etc. Of these, gravity retaining wall is one among the commonly used one. Gravity walls are often suitable for small retaining walls and are not usually built to exceed a height of four feet. This type of retaining wall depends heavily on its self weight to resist the pressure exerted by the materials behind it. They are constructed in such a way that its base is thicker than its top and is often leaning backwards against the materials it is resisting.

The conventional retaining wall design highly depends on experience of engineers, in which structure is designed on the basis of trial and error method. A good design must satisfy the limit states prescribed by specification codes. Attention to various aspects of geotechnical engineering design needs to be considered. But, consideration of all these aspects makes the design complicated and may increase the cost of construction. In order to economize the cost of construction of retaining walls, it needs to vary the dimensions of the wall several times making it very tedious. For this, optimization can be done. Optimization is the process of obtaining the best result under given circumstances. The goal of the optimization is to minimize the total cost or total weight per unit length of the retaining wall subjected to constraints based on stability, bending moment and shear force capacities and all the design requirements. As it is very difficult to obtain such a design satisfying all the safety requirements with the consideration of low cost of construction, it is necessary to cast the problem as an optimization problem.

one of the mathematical programming. MATLAB is one of the efficient numerical methods that can be used for optimization problem.

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Its typical uses include data analysis, exploration, and visualization, scientific and engineering graphics application development, including Graphical User Interface building. MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non interactive language such as C or Fortran.

For the optimization, optimtool can be used in Matlab. Optimization toolbox provides functions for finding parameters that minimize or maximize objectives while satisfying constraints. The toolbox includes solvers for linear programming, mixed-integer linear programming, quadratic programming, nonlinear programming, constrained linear least squares, nonlinear least squares, and nonlinear equations. The optimization problem can be defined with functions and matrices or by specifying variable expressions that reflect the underlying mathematics.

1.2 Objectives

- To develop a program in MATLAB :
- To proportionate gravity retaining wall
- To check safety requirements and optimization of the retaining wall
- To validate the program considering an existing retaining wall

2 LITERATURE REVIEW

Optimum design of retaining walls has been the subject of a number of studies. Some of them are as follows:

M. Ghazavi and V. Salavati (2011) presents in detail the background and implementation of bacterial foraging optimization algorithm (BFOA) suitable for economic optimization and sensitivity analysis of RCC walls. Chemotaxis, swarming, reproduction and elimination-dispersal are the four sequential mechanisms included in the bacterial foraging system. The study showed that the BFOA method can be successfully adopted for the design of RCC retaining walls using minimum cost, overcoming the difficulties associated with the practical and realistic assessment of the structural costs and their complex inter-relationship with the imposed constraints on the solution space.

Song et.al (2013) designed a structural reliability program using Monte Carlo probability theory for gravity retaining wall constructed by new engineering materials. Anti-skid and anti-overturning safety failure mode of retaining wall were considered for the programming of the reliability calculation. The design program diagram of structural reliability indices was created under two failure modes, which are slip failure mode and overturning failure mode.

Sarlbass and Erbatur (2014) have done an optimum design of reinforced concrete cantilever retaining wall. The problem was done as a nonlinear programming problem. Two objective functions- the cost and weight of the wall were considered in the formulation. The behavioral constraints and side constraints for the design were defined as lower and upper limits of the design variables. Using a specially prepared computer program known as RETOPT optimum solution was obtained.

Dembicki and Chi (1989), in their study on the optimization of the geometry of a retaining wall, the effect of interaction between the parametric dimensions of the wall was investigated using a systems approach. The parametric dimensions were considered together as a single unit. Such an analysis provides a more complete mathematical description so that it allows the process of seeking the new geometry of a retaining wall to be automated.

Kaveh and Abadi (2010), for the optimization of retaining wall, they used the improved harmony search where the effects of the improvement on different mathematical functions and optimization problems are illustrated. IHS algorithm is effective at finding areas of the global optimum and is good as other mathematical techniques. The results obtained show that the improved harmony search method is a powerful and an efficient method for finding the optimum solution of structural optimization problems.

Camp and Akin (2012) developed a procedure for designing low-cost or low-weight cantilever reinforced concrete retaining walls, with base shear keys, using big bang-big crunch (BB-BC) optimization. By conducting a series of example design problems, the hybrid BB-BC algorithm showed that it was both computationally efficient and capable of creating low-cost retaining wall designs that satisfy safety, stability, and material constraints. Also sensitivity analysis indicated that formulation of cost is more sensitive to variation in surcharge load, backfill

slope, and the internal angle of friction of the retained soil than the weight model.

V. Nandha Kumar and C.R. Suribabu (2018), using Differential Evolution Algorithm (DEA), conducted a study of weight reduction optimization of reinforced cantilever retaining wall subjected to a sloped backfill. The design variables, constraint equations were determined and optimized with DEA. It was found that structural optimization can be successfully achieved by weight reduction of cantilever retaining wall.

SivakumarBabu and Bhasha (2008) used reliability index to address the uncertainties in soil, concrete, steel, wall proportions and safety for optimal design of cantilever reinforced concrete retaining wall. The study shows that more economical design can be obtained by reduction in the cross sectional area of retaining wall structure through optimization approach brings. As the cross sectional area reduces, there will be reduction in volume of concrete and thereby cost reduction can be found out.

Ahmadi and Varae (2009) used particle swarm algorithm with minimization of cost as objective function to optimize the cantilever reinforced retaining wall and it indicated that there is 12% reduction in concrete volume and 6% for reinforcement.

Keveh and Farhoudi (2011), for the optimal design of cantilever retaining walls, proposed Dolphin Echolocation optimization model. The differential evolution was chosen in order to experiment with all possible design variable combinations as it is significantly faster and robust for solving numerical optimization problems.

3 METHODOLOGY

Generally, the forces acting on the retaining structures are consistent. Overturning, sliding and bearing stress failure modes are mainly considered in the analysis of the retaining structure. Also the structure should be safe against tension failure. The overturning moment about the toe of the wall is balanced by the force due to the active soil pressure of the retained soil weight and the self weight of the structure, the soil above the base, and the surcharge load. For the overturning moment, the passive forces on the front of the toe and the base shear key section are not considered. As per Rankine's theory,

The active earth pressure coefficient K_a is:

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \quad (1)$$

ϕ is the angle of internal friction.

The passive earth pressure coefficient K_p is:

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \quad (2)$$

The factor of safety against overturning about the toe is

defined as:

$$FSO = \frac{\Sigma M_R}{\Sigma M_O} \quad (3)$$

where ΣM_R is the sum of the moments about toe resisting overturning(resisting moments) and ΣM_O is the sum of the moments about toe tending to overturn the structure(overturning moments).

Only the horizontal component of the active force is considered for the sliding mode of failure. Horizontal resisting forces are due to the weight of wall and soil on the base, surcharge load, friction between soil and base of wall, and passive force due to soil on the toe and base shear key sections.

The factor of safety against sliding FSS is defined as:

$$FSS = \frac{\Sigma FR}{\Sigma FD} \quad (4)$$

where ΣFR is the sum of the horizontal resisting forces and ΣFD is the sum of the horizontal sliding forces.

In the bearing analysis of the structure, the base of retaining wall is considered to be a shallow foundation.

The minimum and maximum applied bearing stresses on the base of the foundation are:

$$q_{max} = \frac{\Sigma V}{B} \left\{ 1 \pm \frac{(6 \cdot E)}{B} \right\} \quad (5)$$

$$q_{min} = \frac{\Sigma V}{B} \left\{ 1 \pm \frac{(6 \cdot E)}{B} \right\} \quad (6)$$

where q_{min} and q_{max} are the bearing stresses on the toe and heel sections, B is the width of the base, ΣV is the sum of the vertical forces (due to the weight of wall, the soil above the base, and surcharge load), and E is the eccentricity of the resultant force system expressed as:

$$E = \frac{B}{2} - \frac{\Sigma M_R}{\Sigma V} \quad (7)$$

The eccentricity is determined from the ratio of the summation of overturning moments about the toe to the sum of vertical forces. The factor of safety for the bearing capacity FSB is:

$$FSB = \frac{q_u}{q_{max}} \quad (8)$$

where q_u is the ultimate bearing capacity of the foundation

3.1 Formulation of the design problem

Three mainstages considered in the optimum design of any structure are structural modelling,optimum design modelling and optimization algorithm.In the structural modelling phase,the problem is associated with the determination of

of variables without violating the design constraints.In optimum design modelling the design parameters,vaibles,theconstaints,objective functions etc.For finding optimum, the design begins from a set of parameters and proceeds to the optimum values.

3.1.1Structural modelling

Minimizing the cost of construction and weight of the retaining wall under constraints is the main aim of the optimum design of the retaining wall.

The optimization problem is as follows:

$$\begin{aligned} &\text{minimize } f(X) \\ &\text{subject to} \\ &g_i(X) \leq 0 \quad i=1,2, \dots, p \\ &h_j(X) = 0 \quad j=1,2, \dots, m \\ &L_k \leq X_k \leq U_k \quad k=1, 2 \dots n \end{aligned} \quad (9)$$

where $f(X)$ is the objective function $g_i(X)$, $h_j(X)$ are inequality and equality constraints respectively and L_k , U_k are lower and upper bound constraints. To economic design of retaining wall, the objective function, design variables and design constraints should be defined explicitly.

Optimum Design Modelling

A.Design variables

The program formulation includes mainly five variables representing the dimensions of the gravity retaining wall.They are as follows:

- X_1 = width of the base
- X_2 = projection
- X_3 = thickness at the bottom of the stem
- X_4 =thickness at the top of the stem
- X_5 = thickness of base slab

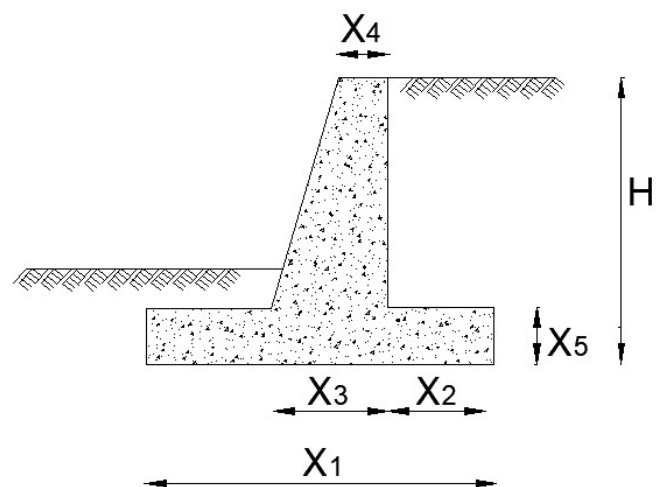


Fig 1.Dimensions of gravity retaining wall

TABLE1: LOWER AND UPPER BOUNDS OF DESIGN VARIABLES

Design variables	Lower bounds	Upper bounds
Width of the base X1	0.5H	0.7H
Toe projection X2	H/6	H/6
Thickness at the bottom of the stem X3	0.3	3
Thickness at the top of the stem X4	0.3	1.5
Thickness of base slab X5	H/10	H/10

H = Height of the retaining wall in metres

Some parameters have to be entered (table 2), so that the proportioning and optimization can be effectively completed. It includes internal angle of friction, unit weight of soil, unit weight of masonry, cost of masonry, coefficient of friction etc.

B.Constraints

The design philosophy for retaining wall deals with the design which provides safety and stability according to the specifications. As per IS 456:2000, the design constraints may be classified as geotechnical and structural requirements. These requirements represent the failure modes as a function of the design variables. Minimum factor of safety coefficients for

TABLE2 :INPUT PARAMETERS FOR DESIGN

Parameter	Unit	Symbol
Internal angle of friction	Degree (°)	ϕ
Height of the stem	Metres (m)	H
Density of soil	kN/m ³	γ_s
Unit weight of masonry	kN/m ³	γ_m
Safe bearing capacity of soil	kN/m ²	SBC
Coefficient of friction under base	mue	μ
Factor of safety against overturning		FS _o
Factor of safety against sliding		FS _s
Factor of safety against bearing capacity failure		FS _B
Cost of masonry	Rs/m ³	C _m

overturning, sliding, and bearing capacity failure modes should be there for feasible retaining wall designs. Failure modes is summarized as in Table 3:

i. Overturning failure mode

The stabilizing moments, due to vertical forces must be greater than the overturning moments, due to horizontal forces to prevent rotation of the wall around its toe. Active earth pressure causes the overturning moments and self weight of the structure provides the stabilizing moments. Overturning failure is a result of excessive lateral earth pressures with relation to retaining wall resistance thereby causing the retaining wall system to overturn.

$$g_1(X) = FS_o - (M_{vtotal}/M_{htotal}) \leq 0 \quad (10)$$

TABLE 3: FAILURE MODES OF RETAINING WALL

Inequality constraints	Failure mode
$g_1(X)$	Overturning stability
$g_2(X)$	Maximum bearing capacity
$g_3(X)$	Minimum bearing capacity
$g_4(X)$	Sliding stability
$g_5(X)$	No tension condition

Where M_{vtotal} = Total vertical moment of forces that tends to resist overturning about toe

M_{htotal} = Total horizontal moment of forces that tends to overturn about toe

FS_o = Factor of safety against overturning

- ii. **Bearing failure mode-** The bearing capacity of the foundation must be large enough to resist the stresses acting along the base of the structure.

$$g_2(X) = P_{max} - S.B.C \leq 0 \quad (11)$$

where S.B.C. = Safe bearing capacity of the soil

P_{max} = Maximum contact pressure at the interface between the wall structure and the foundation soil.

$$g_3(X) = -P_{min} \leq 0 \quad (12)$$

P_{min} = Minimum contact pressure at the interface between the wall structure and the foundation soil.

- iii. **Sliding failure mode-**

The net horizontal forces must be such that the wall is prevented from sliding along its foundation.

Mainly, the lateral earth pressure exerted on the backfill side of the wall provides the sliding force component. Sliding failure is a result of excessive lateral earth pressures with relation to retaining wall resistance thereby causing the retaining wall system to move away (slide) from the soil it retains.

$$g_4(X) = FS_s - ((V_{total} * \mu + \text{Horizontal force from passive pressure}) / H_{total}) \leq 0 \quad (13)$$

where $(V_{total} * \mu + \text{Horizontal force from passive pressure})$ = Resistance to sliding

H_{total} = Total horizontal driving forces.

FS_s = Factor of safety against sliding

- iv. **Tension failure mode-**

For stability, the line of action of the resultant force must lie within the middle third of the base of the foundation.

$$g_5(X) = E - (B/6) \leq 0 \quad (14)$$

Where B = Base width of the wall

E = Eccentricity of the resultant force

C.Objective Function

The objective of design will be minimization of weight or cost or stress concentration factor. Usually, optimization of cost or weight will be the main objective function in structural designs. The forms of the two objective functions for this optimization are consistent, i.e. the cost of concrete or masonry (includes labour cost, installation cost and the cost of the material per unit volume). The cost function f (cost) is:

$$f(\text{cost}) = C_m * V_m \quad (15)$$

where C_m is the unit cost of masonry and V_m is the volume of masonry per unit length of the wall.

The second objective function is based mainly on the weight of the materials. The weight function f (weight) is:

$$f(\text{weight}) = 100 * V_m * \gamma_m \quad (16)$$

where γ_m is the unit weight of masonry and a factor of 100 is used for consistency of units.

In general relation between linear and nonlinear is found to be difficult. The numbers of cases of retaining wall are optimized for one meter length of retaining wall. The design variables described as X_1 is base width of retaining wall, X_2 is toe width of retaining wall, X_3 is bottom thickness of stem, X_4 is top thickness of stem, X_5 is base thickness of retaining wall which describes the shape of the optimum retaining wall.

In addition, all the design variables have lower bounds and upper bounds. In analysis process design considerations are related to total height of retaining wall but in optimization, taking initial assumptions i.e. lower bound and upper bound of the design variables are related to the height of stem and thickness of base is as design variable.

Optimtool is used for the optimization problem. The function for constrained nonlinear minimization is "fmincon". The input arguments are A_{eq} and b_{eq} which are respectively, the coefficients of the linear equality constraints and the corresponding right hand side vector: $A_{eq} * x = b_{eq}$.

3.2 VALIDATION OF THE DESIGN

Details of an existing retaining wall at Ayyapankulam, Thrissur was collected for the validation purposes. It is given in Table 4.

TABLE 4: DETAILS OF RETAINING WALL

Parameter	Values
Density of soil γ_s (kN/m ³)	20
Internal angle of friction ϕ (°)	30
Unit weight of masonry (kN/m ³)	24
Safe bearing capacity of soil (kN/m ²)	250
Height of retaining wall (m)	4
Coefficient of friction under base (μ)	0.5
Factor of safety against overturning	2
Factor of safety against sliding	1.5
Factor of safety for bearing capacity	$P_{max} < SBC, P_{min} > 0$
Cost of masonry (Rs/m ³)	4000

4 RESULTS AND DISCUSSIONS

Matlab is a software package for high-performance numerical computation and visualization. Its built-in-function provide excellent tools for linear algebra computations, data analysis, signal processing etc. The main advantage of Matlab is that user can write his own function in the Matlab language. Other advantages include ease of use, ease of programming, availability of built-in-functions, speed etc.

A program is developed in Matlab for the stability analysis and optimization of gravity retaining wall. The program is used to find the lower bound and upper bound of the variables in the design of retaining wall. Also, the program checks the factor of safety against overturning, bearing capacity failure, sliding and tension failure. Optimization of retaining wall was done using optimtool in the software. In this, optimum cost and optimum weight were given as the objective functions.

Validation is done by considering a gravity retaining wall existing at Ayyappankulam, Thrissur. Its optimum proportioning as per the developed program are given in table 5.

Design variable	Unit	Lower bound	Upper bound	Optimum values for minimum cost	Optimum values for minimum weight
X_1	m	2	2.8	2.6	2.6
X_2	m	0.48	0.68	0.578	0.578
X_3	m	0.3	3	2.41	2.41
X_4	m	0.3	1.5	1.471	1.471
X_5	m	0.48	0.68	0.515	0.515

The values of the behavioural constraints at optimum values of the design variables are given in table 6. It shows that the structure is safe at dimensions obtained at minimum cost and minimum weight.

TABLE6: VALUES OF BEHAVIOURAL CONSTRAINTS AT OPTIMUM VALUES OF DESIGN VARIABLES

Constraints	Minimum cost	Minimum weight
$g_1(X)$	0	0
$g_2(X)$	-195.5688	-195.5688
$g_3(X)$	0.7569	0.6989
$g_4(X)$	0	0
$g_5(X)$	0.0132	0.0109

The optimum value of minimum cost and minimum weight for the retaining wall was also obtained (table 7).

TABLE 5: OPTIMUM VALUES OF DESIGN VARIABLES FOR RETAINING WALL

TABLE7: OPTIMUM VALUES OF OBJECTIVE FUNCTIONS AND VOLUME OF MASONRY

Objective function	Unit	Optimum value	Volume of masonry (m ³ /m)
--------------------	------	---------------	---------------------------------------

Min.cost	Rs/m	32408	8.102
Min.weight	Kg/m	32408	8.102

From the program, lower bound and upper bounds of the variables are obtained. Within these limits, optimum values of dimensions for both minimum weight and minimum cost are obtained without compromising the stability conditions. The values of behavioural constraints shows that the designed dimensions are safe for the retaining wall and are compatible with the stability requirements. Thus validation shows successful design of the program developed in Matlab. Also, optimtool proves to be an easy tool for the optimization of retaining wall.

5 CONCLUSIONS

For achieving economy in conventional analysis without compromising the safety conditions, optimization programming was made by considering geometric constraints.

- Proportioning of the gravity retaining wall was done using Matlab
- A program was developed for finding the optimum dimensions of the retaining wall
- The program was also extended for finding the optimum cost and optimum weight using Optimtool
- Validation was done considering a gravity retaining wall located at Ayyappankulam
- From studying the results, it can be seen that there is a small percentage difference of 4% in base width, 5.07% for thickness at the top of the stem, 4.78% in bottom thickness of the stem and 3% in base slab thickness between original dimensions and dimensions obtained through program.

REFERENCES

- [1]Ahmadi-Nedushan B, Varae H. "Optimal design of reinforced concrete retaining walls using a swarm intelligence technique", First International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering, Civil - Comp Press, Stirlingshire, Scotland, 2009; 26: pp. 1-12.
- [2]Askmarbasl and FuatErbatur (2014) "Optimization and sensitivity of retaining structures" American Society of Civil Engineers (ASCE) J. Geotech. Engg. 1996.122:649-656.
- [3]Charles V. Camp and Alper Akin (2012) "Design of Retaining Walls Using Big Bang-Big Crunch Optimization" American Society of Civil Engineers (ASCE) J. Struct. Eng. 2012.138:438-448.
- [4]EugeniuszDembicki and Tran chi (1989)." System analysis in

calculation of cantilever retaining walls".International journal for numerical and analytical methods in geomechanics, vol. 13, 599-610

[5]Kaveh A, Farhondi N. Dolphin echolocation optimization for design of cantilever walls, Asian J Civil Eng 2016; 17(2): 193-211.

[6]M. Ghazavi, V. Salavati ,(2011) "Sensitivity analysis and design of reinforced concrete cantilever retaining walls using bacterial foraging optimization algorithm" ISGSR ISBN 978-3-939230-01-4

[7]Sable K S and Patil Archana (2012) "Optimization of retaining wall by using optimtool in matlab" International journal of Engineering Research and Technology (IJERT) ISSN :2278 -0181 Vol 1 Issue 6

[8]SivakumarBabu GL, MunwarBasha B. "Optimum design of cantilever retaining walls using target reliability approach", Int J Geomech 2008; 8(4): 240-52.

[9]V. Nandha Kumar and C.R. Suribabu (2018) International Journal Of Optimization In Civil Engineering Int. J. Optim. Civil Eng., 2017; 7(3):433-449

[10]Yunlian Song Jianran and CAO Si LI (2013) "Program Design of Gravity Retaining Wall Structural Reliability with Engineering Materials base on Matlab Software" Advanced Materials Research ISSN: 1662-8985, Vol. 648, pp 166-169

IJSER